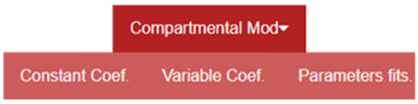
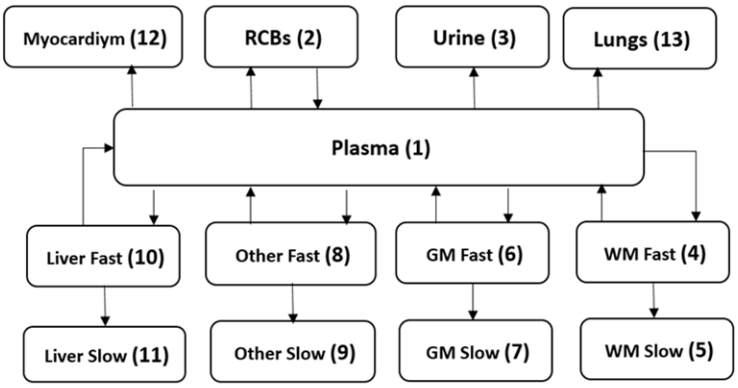
Building a model

Here it is shown how can built and solved a compartmental model.



Consider as an example the case of 18F-FDG (fluorodeoxyglucose) used for positron emission tomography (PET) scan.



{"{Plasma,1}", "{RBCs,2}", "{Urine,3}", "{WhiteMatterFast,4}", "{WhiteMatterSlow,5}", "{GrayMatter-Fast,6}", "{GrayMatterSlow,7}", "{OtherTissueFast,8}", "{OtherTissueSlow,9}", "{LiverFast,10}", "{LiverSlow,11}", "{Myocardium,12}", "{Lungs,13}"}

RBCs–red blood cells; WM–white matter; GM–gray matter

Figure 1.- 18F-FDG compartmental model described in described in EURADOS-Report-2021-04: W. Li, D. Broggio, A. Giussani, T. Vrba, K. Hürkamp, A. Kamp, D. Noßke, L. Struelens, P. Covens: “EURA-DOS Intercomparison on Compartmental Model for 18F-FDG Developed by Hays and Segall”, Neuherberg, July 2021. DOI: 10.12768/zz8y-xw93. (https://eurados.sckcen.be/sites/eurados/files/uploads/Report-Publications/Reports/2021/EURADOS-Report-2021-04.pdf)

- The rate transfer ($k_{i,j}$) from compartment i to compartment j (EURADOS-Report-2021-04 for Geometric mean values) in min^{-1} are:

```
In[∘]:= ratetransferGM =
  {{1, 2, 4.07}, {2, 1, 7.35}, {1, 3, 0.0085}, {1, 4, 0.052}, {4, 1, 0.1},
   {4, 5, 0.042}, {5, 4, 0.0055}, {1, 6, 0.099}, {6, 1, 0.115}, {6, 7, 0.059},
   {7, 6, 0.0066}, {1, 8, 0.348}, {8, 1, 0.097}, {8, 9, 0.015}, {1, 10, 0.038},
   {10, 1, 0.186}, {10, 11, 0.006}, {1, 12, 0.003}, {1, 13, 0.0016}};
```

```
In[ ]:= TableForm[ratetransferGM]
Out[ ]//TableForm=
```

1	2	4.07
2	1	7.35
1	3	0.0085
1	4	0.052
4	1	0.1
4	5	0.042
5	4	0.0055
1	6	0.099
6	1	0.115
6	7	0.059
7	6	0.0066
1	8	0.348
8	1	0.097
8	9	0.015
1	10	0.038
10	1	0.186
10	11	0.006
1	12	0.003
1	13	0.0016

- The half - life (for ^{18}F= 109.77 min^{-1}

```
decayconstant = Log[2] / 109.77;
```

Example : Injection of 1 Bq in compartment 1 (Blood) a t = 0, being 0 in the rest of compartments the initial condition.

- Then the initial condition is:

```
initialcontidion = {1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0};
```

- Input function in each compartment for t > 0 (Note that you can used other kind of inputs such us: {3 Exp[-0.3], 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}) :

```
inputs = {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0};
```

- Times ti to compute the accumulated disintegrations in each compartments (in this case is in min to match with the k_{ij} values):

```
times = {0.1, 1.0, 10, 100, 1000};
```

To solve the problem these data are given as input at :

http://oed.usal.es/webMathematica/Biokmod/ →Compartmental-> Mod→Constant Coef.

http://oed.usal.es/webMathematica/Biokmod/biokmod1.jsp

- Input

Select time unit used for t (decay constant, rate transfer coeffs (kij), representations, ...) : min
Rate transfers amongs compartments:

```
{{1, 2, 4.07}, {2, 1, 7.35}, {1, 3, 0.0085}, {1, 4, 0.052}, {4, 1,
0.1}, {4, 5, 0.042}, {5, 4, 0.0055}, {1, 6, 0.099}, {6, 1, 0.115}, {6,
7, 0.059}, {7, 6, 0.0066}, {1, 8, 0.348}, {8, 1,  0.097}, {8, 9,
0.015}, {1, 10, 0.038}, {10, 1, 0.186}, {10, 11,   0.006}, {1, 12,
0.003}, {1, 13, 0.0016}}
```

Number of compartments: 13 Decay constant: Log[2]/109.77

Initial conditions at time t = 0: {1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

Input function in each compartment : {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

All values of this field must be {0,.., 0} if it is an impulsive single-input, because the inputs are the initial conditions.

Time t to evaluate the content in each compartment (i.e.: t or {5, 20, 30}) : t

Range of t to be plotted: From t-min 0 to t-max 500

Time t to compute the acumulated disintegrations in each compartment : {0.1, 1.0, 10, 100, 1000}

Evaluate

- Output

Differential equation

$$x_1'(t) = -4.62641\, x_1(t) + 7.35\, x_2(t) + 0.1\, x_4(t) + 0.115\, x_6(t) + 0.097\, x_8(t) + 0.186\, x_{10}(t)\cdot$$
$$x_2'(t) = 4.07\, x_1(t) - 7.35631\, x_2(t) + 0.$$
$$x_3'(t) = 0.0085\, x_1(t) - 0.00631454\, x_3(t) + 0.$$
$$x_4'(t) = 0.052\, x_1(t) - 0.148315\, x_4(t) + 0.0055\, x_5(t) + 0.$$
$$x_5'(t) = 0.042\, x_4(t) - 0.0118145\, x_5(t) + 0.$$
$$x_6'(t) = 0.099\, x_1(t) - 0.180315\, x_6(t) + 0.0066\, x_7(t) + 0.$$
$$x_7'(t) = 0.059\, x_6(t) - 0.0129145\, x_7(t) + 0.$$
$$x_8'(t) = 0.348\, x_1(t) - 0.118315\, x_8(t) + 0.$$
$$x_9'(t) = 0.015\, x_8(t) - 0.00631454\, x_9(t) + 0.$$
$$x_{10}'(t) = 0.038\, x_1(t) - 0.198315\, x_{10}(t) + 0.$$
$$x_{11}'(t) = 0.006\, x_{10}(t) - 0.00631454\, x_{11}(t) + 0.$$
$$x_{12}'(t) = 0.003\, x_1(t) - 0.00631454\, x_{12}(t) + 0.$$
$$x_{13}'(t) = 0.0016\, x_1(t) - 0.00631454\, x_{13}(t) + 0.$$
$$x_1(0) = 1$$
$$x_2(0) = 0$$
$$x_3(0) = 0$$
$$x_4(0) = 0$$
$$x_5(0) = 0$$
$$x_6(0) = 0$$
$$x_7(0) = 0$$
$$x_8(0) = 0$$
$$x_9(0) = 0$$
$$x_{10}(0) = 0$$
$$x_{11}(0) = 0$$
$$x_{12}(0) = 0$$
$$x_{13}(0) = 0$$

- Compartment content (Bq).

The solution are the impulse response functions because it has been used a single input 1, Out[∘]=

$$\{x_1[t] \to 0.379079\, e^{-11.6304\, t} + 0.454068\, e^{-0.465429\, t} + 0.00186102\, e^{-0.194332\, t} + 0.00885244\, e^{-0.170587\, t} + 0.00321481\, e^{-0.14548\, t} + 0.134277\, e^{-0.0278066\, t} + 0.000254002\, e^{-0.0103031\, t} + 0.0183932\, e^{-0.00835781\, t},$$

$$x_2[t] \to -0.360976\, e^{-11.6304\, t} + 0.268189\, e^{-0.465429\, t} + 0.00105758\, e^{-0.194332\, t} + 0.00501403\, e^{-0.170587\, t} + 0.00181453\, e^{-0.14548\, t} + 0.0745729\, e^{-0.0278066\, t} + 0.000140728\, e^{-0.0103031\, t} + 0.0101879\, e^{-0.00835781\, t},$$

$$x_3[t] \to -0.000277197\, e^{-11.6304\, t} - 0.00840657\, e^{-0.465429\, t} - 0.0000841344\, e^{-0.194332\, t} - 0.000458054\, e^{-0.170587\, t} - 0.000196355\, e^{-0.14548\, t} - 0.053106\, e^{-0.0278066\, t} - 0.000541309\, e^{-0.0103031\, t} - 0.0765157\, e^{-0.00835781\, t} + 0.139585\, e^{-0.00631454\, t},$$

$$x_4[t] \to -0.00171677\, e^{-11.6304\, t} - 0.0745772\, e^{-0.465429\, t} - 0.00216246\, e^{-0.194332\, t} - 0.0221122\, e^{-0.170587\, t} + 0.0366417\, e^{-0.14548\, t} + 0.0517397\, e^{-0.0278066\, t} - 0.000891351\, e^{-0.0103031\, t} + 0.0130786\, e^{-0.00835781\, t},$$

$$x_5[t] \to 6.20594 \times 10^{-6}\, e^{-11.6304\, t} + 0.00690507\, e^{-0.465429\, t} + 0.000497616\, e^{-0.194332\, t} + 0.00584932\, e^{-0.170587\, t} - 0.0115134\, e^{-0.14548\, t} - 0.135884\, e^{-0.0278066\, t} - 0.0247681\, e^{-0.0103031\, t} + 0.158908\, e^{-0.00835781\, t},$$

$$x_6[t] \to -0.00327761\, e^{-11.6304\, t} - 0.158143\, e^{-0.465429\, t} - 0.0155208\, e^{-0.194332\, t} + 0.071853\, e^{-0.170587\, t} + 0.00842609\, e^{-0.14548\, t} + 0.0744079\, e^{-0.0278066\, t} + 0.00120311\, e^{-0.0103031\, t} + 0.021051\, e^{-0.00835781\, t},$$

$$x_7[t] \to 0.0000166455\, e^{-11.6304\, t} + 0.020619\, e^{-0.465429\, t} + 0.00504762\, e^{-0.194332\, t} - 0.0268869\, e^{-0.170587\, t} - 0.00375013\, e^{-0.14548\, t} - 0.294793\, e^{-0.0278066\, t} + 0.0271811\, e^{-0.0103031\, t} + 0.272566\, e^{-0.00835781\, t},$$

$$x_8[t] \to -0.0114592\, e^{-11.6304\, t} - 0.455226\, e^{-0.465429\, t} - 0.00851962\, e^{-0.194332\, t} - 0.0589342\, e^{-0.170587\, t} - 0.0411823\, e^{-0.14548\, t} + 0.516291\, e^{-0.0278066\, t} + 0.000818364\, e^{-0.0103031\, t} + 0.0582123\, e^{-0.00835781\, t},$$

$$x_9[t] \to 0.0000147872\, e^{-11.6304\, t} + 0.014873\, e^{-0.465429\, t} + 0.000679695\, e^{-0.194332\, t} + 0.00538138\, e^{-0.170587\, t} + 0.00443883\, e^{-0.14548\, t} - 0.360337\, e^{-0.0278066\, t} - 0.0030777\, e^{-0.0103031\, t} - 0.427347\, e^{-0.00835781\, t} + 0.765373\, e^{-0.00631454\, t},$$

$$x_{10}[t] \to -0.00126005\, e^{-11.6304\, t} - 0.0645962\, e^{-0.465429\, t} + 0.0177556\, e^{-0.194332\, t} + 0.0121322\, e^{-0.170587\, t} + 0.00231219\, e^{-0.14548\, t} + 0.0299255\, e^{-0.0278066\, t} + 0.0000513377\, e^{-0.0103031\, t} + 0.00367948\, e^{-0.00835781\, t},$$

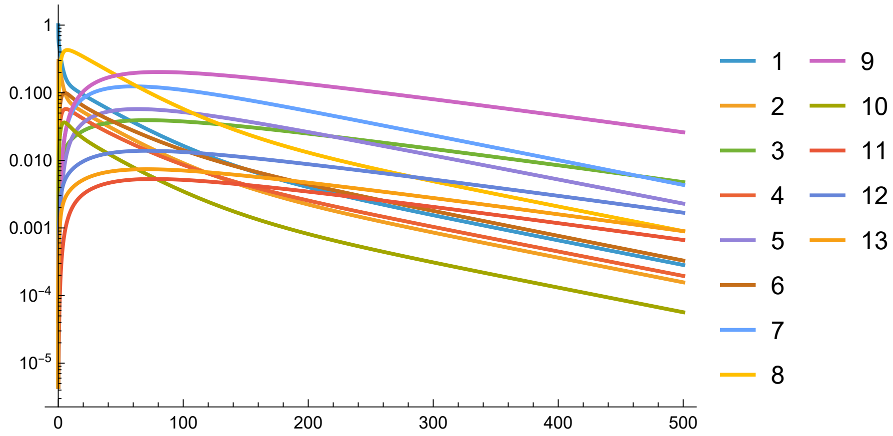
$$x_{11}[t] \to 6.50397 \times 10^{-7}\, e^{-11.6304\, t} + 0.000844183\, e^{-0.465429\, t} - 0.000566616\, e^{-0.194332\, t} - 0.000443123\, e^{-0.170587\, t} - 0.000099688\, e^{-0.14548\, t} - 0.00835439\, e^{-0.0278066\, t} - 0.0000772283\, e^{-0.0103031\, t} - 0.0108047\, e^{-0.00835781\, t} + 0.0195009\, e^{-0.00631454\, t},$$

$$x_{12}[t] \to -0.0000978344\, e^{-11.6304\, t} - 0.00296702\, e^{-0.465429\, t} - 0.0000296945\, e^{-0.194332\, t} - 0.000161666\, e^{-0.170587\, t} - 0.0000693017\, e^{-0.14548\, t} - 0.0187433\, e^{-0.0278066\, t} - 0.00019105\, e^{-0.0103031\, t} - 0.0270056\, e^{-0.00835781\, t} + 0.0492654\, e^{-0.00631454\, t},$$

$$x_{13}[t] \to -0.0000521783\, e^{-11.6304\, t} - 0.00158241\, e^{-0.465429\, t} - 0.0000158371\, e^{-0.194332\, t} - 0.0000862219\, e^{-0.170587\, t} - 0.0000369609\, e^{-0.14548\, t} - 0.00999642\, e^{-0.0278066\, t} - 0.000101893\, e^{-0.0103031\, t} - 0.014403\, e^{-0.00835781\, t} + 0.0262749\, e^{-0.00631454\, t}\}$$

- Time activity curves (TAC)

  Note The graphic represents the fractional activity in each compartment because it has been used a single input 1, Out[ ]=



- Time integrated activity (Bq s)

Note. Here is included in column 1 the Time integrated activity coefficient's (TIAC) because it has been used a single input 1. TIAC = TIA (1000 min)/60 (min/h)

	TIAC (h)	0.1 min (Bqs)	1. min (Bqs)	10. min (Bqs)	1000. min (Bqs)
1	0.135768	5.00566	33.5678	145.058	488.735
2	0.075116	0.847719	16.4691	79.6844	270.4
3	0.182758	0.0022443	0.156605	7.70174	655.656
4	0.0548293	0.0136632	0.911708	29.2373	197.364
5	0.194915	0.0000197018	0.0136944	5.22847	701.431
6	0.0895101	0.0259842	1.71676	50.8097	322.201
7	0.408926	0.0000526472	0.0363174	13.0179	1471.67
8	0.399337	0.0915322	6.16497	213.987	1437.51
9	0.948613	0.0000471319	0.0330307	13.5712	3402.56
10	0.0260152	0.00996758	0.654882	18.4878	93.6485
11	0.0247193	2.05446×10^{-6}	0.00141359	0.498234	88.6725
12	0.0645027	0.000792104	0.0552724	2.71826	231.408
13	0.0344014	0.000422456	0.0294786	1.44974	123.418

These values are in agreement with EURADOS Intercomparison on Compartmental Model for 18 F - FDG Developed by Hays and Segall. (Table2